Technical University of Denmark

Lifetime as related to materials quality with special reference to wood

Nielsen, Lauge Fuglsang

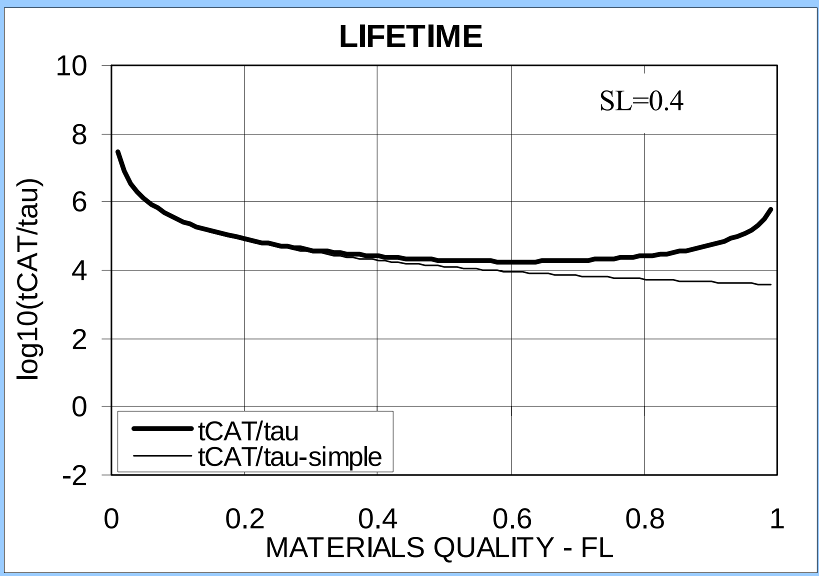
Publication date:
2004

Document Version
Publisher's PDF, also known as Version of record

Lifetime as related to materials quality

with special reference to wood

Lauge Fuglsang Nielsen



DANMARKS TEKNISKE UNIVERSITET

BYG•DTU R-105
November 2004
ISSN 1601-2917
ISBN 87-7877-171-4

Lifetime as related to materials quality

with special reference to wood

Lauge Fuglsang Nielsen

Abstract: It is demonstrated in this report, how the lifetime and residual strength behavior of viscoelastic materials can be analyzed by the so-called DVM-theory (Damaged Viscoelastic Material).

Formulas to be used are presented in an appendix at the end of this report. The results of two theory-versions are considered: An accurate version and a simplified version, which applies accurately at low material qualities.

To demonstrate the power of a DVM analysis it has been chosen to consider the long-time behavior of wood. Results are presented graphically as they are developed from the basic mathematical expressions, originally developed in (T1,2,3), further developed in (4,5,6), and presented in the appendix.

Wood has been chosen because this material is an excellent example of viscoelastic materials containing damages of various kinds ('structural wood' with knots, 'clear wood' with smaller defects). It is very likely that conclusions drawn for wood behavior can be generalized to the behavior of other similar materials or material structures such as fiber-reinforced epoxy, finger joints, other connectors, and concrete.

Contents

Introduction

The influence of wood quality on lifetime has been the subject of almost religious feelings since Borg Madsen (7) suggested the idea that structural wood (commercial timber) is "tougher" (lives longer) than clear (defect free) wood when loaded to the same fraction of respective strengths. There are Madsen-believers and there are

Madison-believers. The latter part considers wood quality to be of no significant influence on lifetime, which remains as observed in the well-known Madison experiments on clear wood, Wood (8). Both parts refer to experimental evidence to prove their views, e.g. Madsen and Johns (9), Spencer (10), Foshi and Barrett (11,12), Krebs (13), Gerhards (14), Hoffmeyer (15).

It is obvious that the two views are inconsistent considering the whole spectrum of wood qualities. Neither one nor the other can be trusted as a reliable basis of establishing general quality-lifetime rules (codes) for practice. The total amount of experimental evidence can only be explained consistently by considering each view to cover separate and limited ranges of wood quality.

It is demonstrated in (4,5) that the author's DVM-model (*Damaged Viscoelastic Material*) has the potential of describing this feature. Examples are presented which clearly suggest that the two "believes" previously referred to might very well be reflections of the same physical phenomenon, namely damage (or crack) propagation in viscoelastic materials. Quoting Borg Madsen: 'Structural wood and clear wood is as different as concrete and cement'. Borg Madsen is right. His brave statement, however, should not be misunderstood. It does not exclude the possibility of a basic composite model, which behaves differently, depending on the content of disturbing elements (such as cracks).

In a generalized form the DVM theory is developed for the analysis of viscoelastic materials and structural elements (4,5,6). As such it can be expected to apply for a number of materials such as timber, clear wood (parallel and perpendicular to grain), Glu-lam, particle- and hardboard, fiber-reinforced epoxy, concrete, finger joints, and other connectors.

This observation is worth noticing when code strategies are planned for the application of new, probably high quality, wood based materials. One cannot totally rely on 'old' empirical knowledge.

Finally, the results of the DVM theory are presented dimensionless. Thus, the solutions are 'global' with respect to humidity for example, which can be considered as explained in (16).

As previously indicated the basic results of the DVM theory are valid for any viscoelastic material (4,5), including concrete (17,18). In the present note, however, all expressions are specialized on wood behavior, meaning that a Power-Law creep behavior is assumed as developed in (19).

Wood quality

Wood quality is defined as immediate strength, σ_{CR}, relative to theoretical strength, σ_L. We introduce the symbol $FL = \sigma_{CR}/\sigma_L$ for strength level – or wood quality. The theoretical strength cannot be kept due to damages of sizes, l. Large damages such as knots reduce strength more than small damages in clear wood.

The quality graph in Figure 1 is obtained from Equation 4 in the appendix introducing a reference strength level of $FL_1 = 0.8$ and a damage ratio of $\kappa = l/d$ where l is damage size, while d is the size of a damage nucleus.

This procedure follows a suggestion made in (4,5) where a damage nucleus of d = 0.3 mm is estimated. We notice that the above mentioned reference strength is obtained when the damage size, l, equals the damage nucleus, d.

Remark: Strength levels of FL > 0.8 can only be obtained improving the microstructure of wood decreasing the size of damage nucleus.

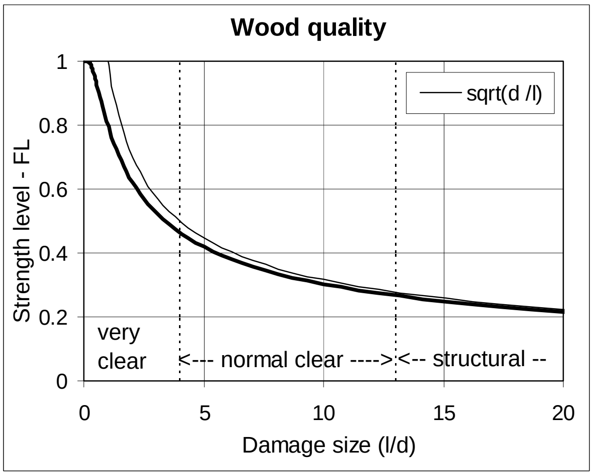


__Figure 1.__ Wood quality estimated from damage size, l, relative to the damage nucleus d = 0.3 mm. Strength levels of FL > 0.8 can only be obtained improving the microstructure of wood by decreasing the size of the damage nucleus.

Estimate

An easy way of estimating materials quality (strength level) is by the following very accurate general approximation, also presented in (4,5) and in (20). The damage nucleus is related to traditional material parameters as shown in Equation 4a of the appendix.

$$FL \approx (1 - \exp^{-d/\ell})^{1/2} \qquad \to \quad (d/\ell)^{1/2} \quad \text{for} \quad \ell/d > 10$$

Lifetime and residual strength

Equations 10, 12, and 5 in the appendix predict these properties. The residual strength (ratio), S_R, is time dependent strength relative to strength at time t = 0. Time is normalized with respect to the relaxation time, τ, in the Power-Law creep description; see Equation 6 in the appendix. For a constant load level of SL = 0.5 the two properties are illustrated in Figures 2 – 4. SL is load relative to wood strength at age of loading, t = 0. The relaxation time of the Power-Law assumed is denoted by τ. For notations in general, see list of notations at the end of this note.

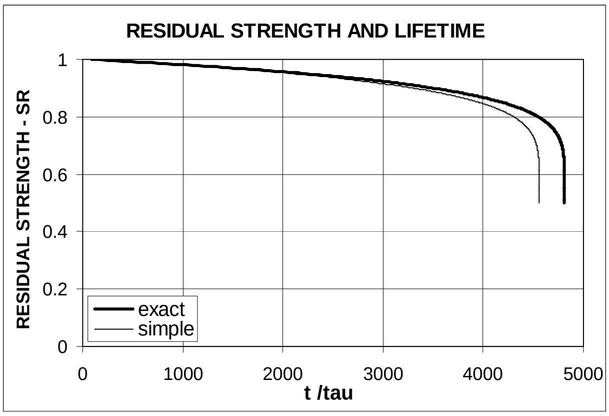


__Figure 2.__ Residual strength and lifetime of low-quality wood (FL = 0.2). Load level is SL = 0.5. Power-Law creep with relaxation time, τ, and a creep power of b=0.25.

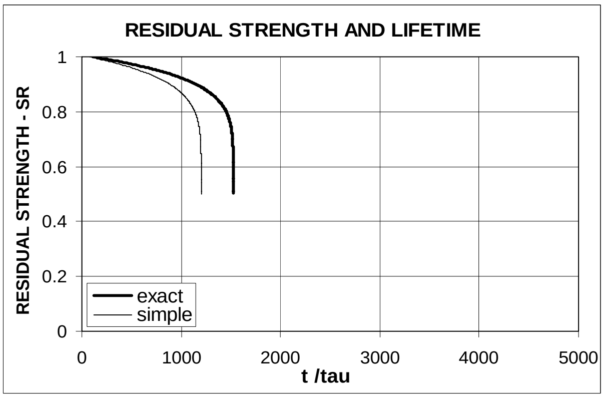


Figure 3. *Residual strength and lifetime of medium quality wood (FL = 0.4). Load level is SL = 0.5. Power-Law creep with relaxation time, τ, and a creep power of b=0.25.*

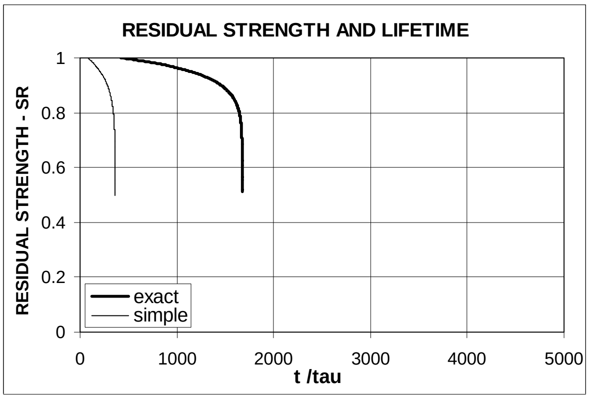


Figure 4. *Residual strength and lifetime of high quality wood (FL = 0.8). Load level is SL = 0.5. Power-Law creep with relaxation time, τ, and a creep power of b=0.25.*

Lifetime versus wood quality

Equations 10 and 12 in the appendix predict these quantities as illustrated in Figures 5 – 7 for three load levels, SL = 0.2, 0.4, and 0.8. Time to catastrophic failure is denoted by t_{CAT}. Abbreviations introduced have the meanings previously explained; see also the list of notations at the end of this note.

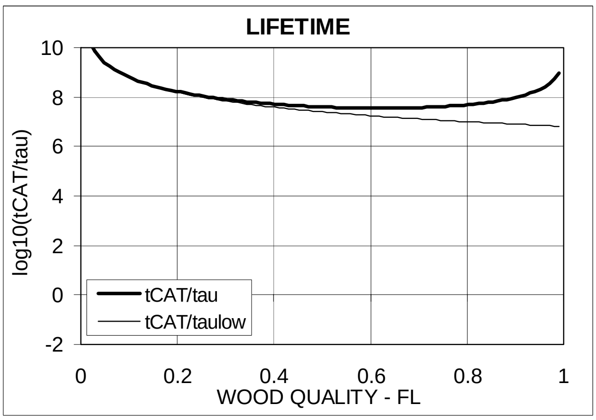


Figure 5. *Lifetime for various wood qualities. b = 0.25 and SL = 0.2.*

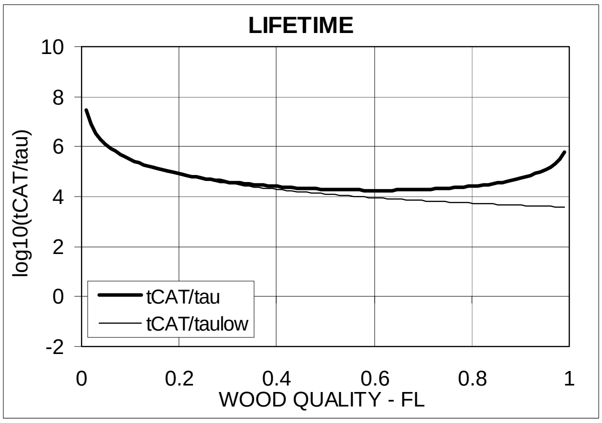


Figure 6. *Lifetime for various wood qualities. b = 0.25, and SL = 0.4.*

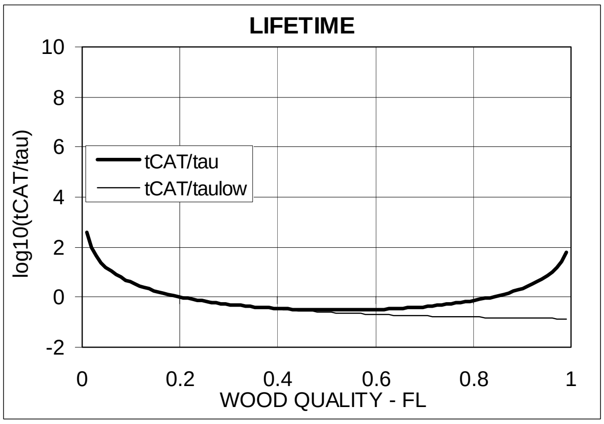


Figure 7. *Lifetime for various wood qualities. b = 0.25 and SL = 0.8.*

Discussion

Figures 2 – 7 demonstrate that low quality structural wood and high quality clear wood lives longer than normal clear wood when loaded to the same fraction of the respective strengths. At middle range qualities, lifetime is practically not influenced by quality.

Strength and lifetime distributions

Wood is not a homogeneous material in the sense that members of a group are identical. Every member has an individual damage structure, which clearly defines its position in a characteristic strength distribution. Thus, when real wood is considered as a group, lifetime is a result of strength distribution, and is therefore itself given by a distribution. Based on the DVM-lifetime analysis (as compiled in the appendix) this phenomenon has been studied by the author in (21). Some results are shown in Figures 8 and 9 reproduced from (4).

It was demonstrated in (21) that the two distributions considered are correlated, which means that lifetime distribution can be predicted from strength distribution. At the same time a discussion was made on how to extract a maximum of lifetime information from experimental data.

Another example of a DVM-damage analysis of wood, where strength distribution has been considered, is illustrated in Figure 10, also reproduced from (4). The experimental data are from (22): Two groups of specimens from a population of wood were loaded to a constant load level. The experiments were stopped when a certain fraction of the population had failed (approximately 30% and 90%). Then the (residual) strength of the rest of the groups was measured. As can be seen from Figure 10, there is an excellent agreement between measured strength and strength theoretically predicted.

Comments

Any realistic reliability study of structures requires information on the reliability of the building materials being used. A reliability analysis of a wood structure, for example, cannot be made without knowledge of the strength and lifetime distributions of the structural wood material. Normally, purely empirical distributions are chosen for each event without any guaranty that this procedure will not introduce inconsistency, which may influence the results of a reliability analysis in some unknown way.

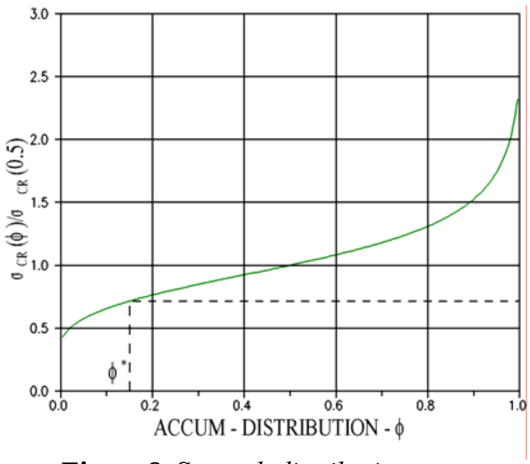


Figure 8. *Strength distribution.*

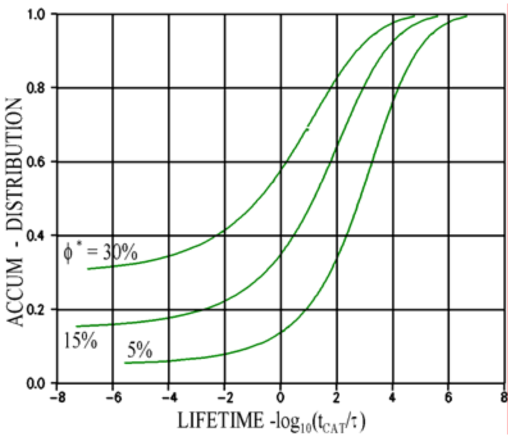


Figure 9. *Lifetime distribution associated with strength distribution from Figure 8. Load is $\sigma = \sigma_{CR}(\varphi^{*})$. FL = FL(0.5) = 0.2.*

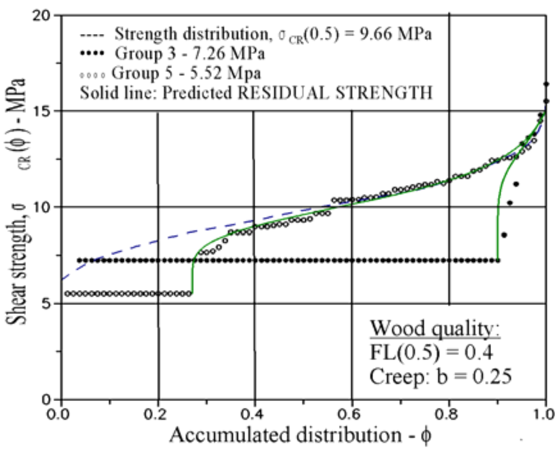


Figure 10. *Residual strength distribution in duration of shear load experiments (22) on small clear wood specimens. Horizontal string represents no residual strength.*

Conclusions and final remarks

- The immediate statement of Borg Madsen's holds for wood of quality FL < approximately 0.5: Lifetime decreases with increasing wood quality. For FL > 0.5, however, lifetime increases.
- A simple lifetime analysis becomes more and more conservative for increasing wood quality FL > 0.5, meaning that too low lifetimes are predicted.
- Lifetime and residual strength can be determined simultaneously; meaning that evaluation of recycled wood becomes possible.

- The results presented are dimensionless, meaning that they are 'global' with respect to moisture for example.
- Strength and lifetime distributions can be correlated.

Loading mode: It is obvious that the DVM-theory, as described in this report, applies immediately in the analysis of structures where loading is 'force controlled'. The analysis of structures with 'deformation controlled' loadings, however, can also be considered. Some minor modifications, however, have to be introduced, see (23).

Appendix

Damaged viscoelastic materials

Prediction of lifetime behaviour

A compilation of formulas

Lauge Fuglsang Nielsen

This appendix is prepared for exercises in lifetime analysis of viscoelastic building materials. It is formed as a brief operational summary of expression originally developed by the author in (1,2). Symbols used in general are explained at the end of this report. More special symbols and symbol-combinations (A, B, and D), however, have been slightly 'modernized' to be consistent with such used in more recent papers (4,5).

It is strongly recommended to scan the general list of notations before studying the content of the appendix. In this context it is also useful to study Figure 11, which shows the basic materials Dugdale model (24) used to develop the expressions presented. The text is rather brief without much text and explanations. A full operational understanding of the appendix can only be achieved applying it on examples such as wood considered in the main text of this report.

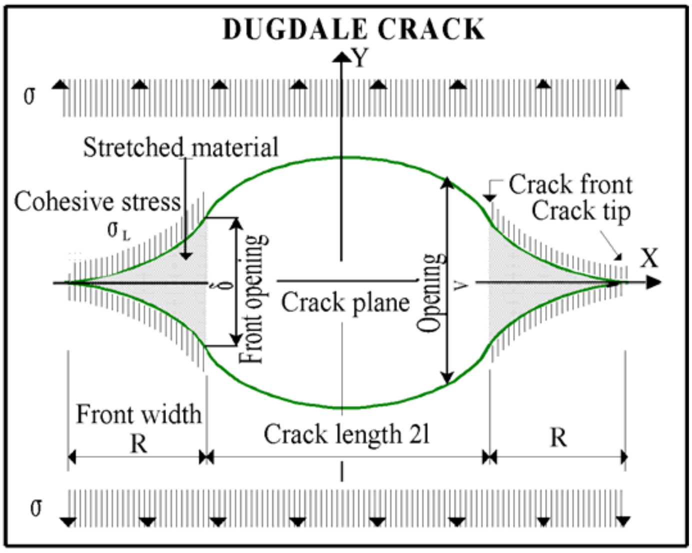


Figure 11. *The so-called Dugdale model, which is the basic model, used for the expressions presented in this appendix. The failure criterion for the model is that the crack front opening, δ, becomes critical, meaning* $\delta = \delta_{CR}$.

Strength and damage

Strength of a damaged material can be expressed by Equation 1 reproduced from (1,2) with symbols explained in the list of symbols at the end of this note. The failure criterion for the Dugdale model shown in Figure 11 is that the crack front opening, δ, becomes critical, meaning $\delta = \delta_{CR}$ (or strain energy release rate, $\Gamma = \sigma_L\delta$, becomes critical, $\Gamma_{CR} = \sigma_L\delta_{CR}$).

$$\sigma_{CR}(\ell) = \frac{2\sigma_L}{\pi}\arccos\left(\exp\left(-\frac{\pi E\Gamma_{CR}}{8\ell\sigma_L^2}\right)\right) \quad \text{with } \Gamma_{CR} = \sigma_L\delta_{CR} \text{ (crit. strain energy release rate)} \quad (1)$$

We can solve this expression with respect to the critical strain energy release rate. We get Equation 2, which can be used, in experimental damage analysis.

$$\Gamma_{CR} = -\frac{8\ell_1\sigma_L^2}{\pi E}\log_E\cos\left(\frac{\pi}{2}\frac{\sigma_{CR}(\ell_1)}{\sigma_L}\right) \quad \text{at references} \begin{cases}\text{strength } \sigma_{CR}(\ell_1)\\ \text{damage size } \ell_1\end{cases} \quad (2)$$

Now strength at two damage sizes can be related as follows

$$\frac{\sigma_{CR}(\ell)}{\sigma_L} = \frac{2}{\pi}\arccos\left(\exp\left(\frac{\ell_1}{\ell}\log_E\cos\left(\frac{\pi}{2}\frac{\sigma_{CR}(\ell_1)}{\sigma_L}\right)\right)\right) = \frac{2}{\pi}\arccos\left(\left(\cos\left(\frac{\pi}{2}\frac{\sigma_{CR}(\ell_1)}{\sigma_L}\right)\right)^{\ell_1/\ell}\right) \quad (3)$$

An immediate consequence of this expression is a similar relation between strength levels,

$$FL(\ell) = \frac{2}{\pi}\arccos\left[\left(\cos\left(\frac{\pi}{2}FL(\ell_1)\right)\right)^{1/\kappa}\right] \quad \text{with damage ratio: } \quad \kappa = \ell/\ell_1 \quad (4)$$

Strength at two different damage sizes can now be related by the so-called residual strength ratio, S_R,

$$\text{Residual strength ratio: } S_R = \frac{\sigma_{CR}(\ell)}{\sigma_{CR}(\ell_1)} = \frac{FL(\ell)}{FL(\ell_1)} \to \frac{1}{\kappa^{1/2}} \text{ for } FL(\ell) \to 0 \quad (5)$$

Strength estimate

The following expression, suggested in (5,20), can be used to estimate strength level from a so-called damage nucleus, d. This relation is a very accurate approximation of Equation 4 with $FL(\ell_1) = 0.8$ and $\kappa = \ell/d$.

$$FL \approx (1-\exp^{-d/\ell})^{1/2} \quad \to \quad (d/\ell)^{1/2} \quad \text{for } \ell/d > 10 \quad \text{where}$$
$$d = \frac{E\Gamma_{CR}}{\pi\sigma_L^2} \quad \text{is damage nucleus} \approx \begin{cases}\text{average center distance beteween the}\\ \text{stronger (or weeker) parts of microstructure}\end{cases} \quad (4a)$$

Lifetime

The lifetime expressions presented in this section are from (2,4,5). They apply for viscoelastic materials exhibiting creep of the Power-Law type (19)[1] shown in Equation 6. Strength level FL refers to reference state at t = 0.

1) The original expressions in (1,2) apply for arbitrary creep functions

Creep function

$$C = 1 + \left(\frac{t}{\tau}\right)^b \qquad \text{Power} - \text{Law creep} \qquad (6)$$

where t, b, and τ denote time, creep power, and relaxation time respectively

Damage propagation

$$\frac{d\kappa}{dt} = \frac{D}{qB}\frac{\kappa/\tau}{(AB/\kappa - 1)^{1/b}} \qquad \text{(arbitrary wood quality, FL)} \qquad (7)$$

$$\frac{d\kappa}{dt} = \frac{(\pi FL * SL)^2}{8q}\frac{\kappa/\tau}{(1/(\kappa SL^2) - 1)^{1/b}} \qquad \text{(lower wood quality, FL} < 1/3) \qquad (8)$$

The abbreviations appearing in Equations 7 and 8 have the following meanings.

$$\begin{aligned}
q &= \left[\frac{(1+b)(2+b)}{2}\right]^{1/b} \\
A &= \frac{\log_E(\cos(\frac{\pi}{2}FL))}{\log_E(\cos(\frac{\pi}{2}FL * SL))} && \Rightarrow \frac{1}{SL^2} && \text{for } FL \to 0 \qquad (9) \\
B &= \beta + (1-\beta)\cos\left(\frac{\pi}{2}FL * SL\right) \quad (\text{with } \beta = \frac{4-b}{5}) && \Rightarrow 1 && \text{for } FL \to 0 \\
D &= 1 - \cos\left(\frac{\pi}{2}FL * SL\right) && \Rightarrow \frac{\pi^2}{8}(FL * SL)^2 && \text{for } FL \to 0
\end{aligned}$$

Deadload lifetime

For a constant load Equations 7 and 8 can be solved to obtain the following lifetime expressions.

Arbitrary materials quality, FL

$$\begin{aligned}
t_S &= (A-1)^{1/b} && \text{time to start of damage propagation} \\
\frac{t}{\tau} &= (A-1)^{1/b} + \frac{Bq}{D}\left(H(AB) - H\left(\frac{AB}{\kappa}\right)\right) && \text{time to develop damage ratio } \kappa \qquad (10) \\
\frac{t_{CAT}}{\tau} &= (A-1)^{1/b} + \frac{Bq}{D}H(AB) && \text{time to catastrophic failure } (\kappa_{CR} = AB)
\end{aligned}$$

where the function H = H(U) is given by

$$\begin{aligned}
H(U) &= \frac{(U-1)^3}{3} - \frac{(U-1)^2}{2} + (U-1) - \log_E(U) && (b = 1/3) \\
H(U) &= \frac{(U-1)^4}{4} - \frac{(U-1)^3}{3} + \frac{(U-1)^2}{2} - (U-1) + \log_E(U) && (b = 1/4) \qquad (11) \\
H(U) &= \frac{(U-1)^5}{5} - \frac{(U-1)^4}{4} + \frac{(U-1)^3}{3} - \frac{(U-1)^2}{2} + (U-1) - \log_E(U) && (b = 1/5) \\
&\underline{H(U) \equiv 0 \quad \text{when} \quad U \le 1}
\end{aligned}$$

Low materials quality, FL < 1/3

$$t_S = \left(\frac{1}{SL^2} - 1\right)^{1/b} \quad \text{time to start of damage propagation}$$

$$\frac{t}{\tau} = \left(\frac{1}{SL^2} - 1\right)^{1/b} + \frac{8q}{(\pi FL * SL)^2}\left(H\left(\frac{1}{SL^2}\right) - H\left(\frac{1}{\kappa SL^2}\right)\right) \quad \text{time to develop damage ratio } \kappa \tag{12}$$

$$\frac{t_{CAT}}{\tau} = \left(\frac{1}{SL^2} - 1\right)^{1/b} + \frac{8q}{(\pi FL * SL)^2} H\left(\frac{1}{SL^2}\right) \quad \text{time to catastrophic failure } \left(\kappa_{CR} = \frac{1}{SL^2}\right)$$

where the function H = H(U) is kept from Equation 11.

List of symbols

Subscripts	
Initial, reference, state (at time 0) (omitted if obvious from text)	1
General	
Theoretical strength	σ_L
Real strength (at time 0)	σ_{CR}
Strength level (Materials quality)	$FL = \sigma_{CR}/\sigma_L$
Load	σ
Load level	$SL = \sigma/\sigma_{CR}$
Young's modulus	E
Creep	
Time in general	t
Creep power	b (for wood, $b \approx 1/5 - 1/3$)
Relaxation time in damage area	τ
Defects	
Critical damage opening	δ_{CR}
Critical strain energy release rate	$\Gamma_{CR} = \sigma_L \delta_{CR}$
Damage size (or crack length)	2ℓ
Damage ratio	$\kappa = \ell/\ell_1$
Residual strength	$S_R = \sigma_{CR}(\ell)/\sigma_{CR}(\ell_1)$
Time to start of damage propagation	t_S
Time to catastrophic failure	t_{CAT}

Literature

---

1) Nielsen, L. Fuglsang: "Crack Propagation in Linear-Viscoelastic Materials", Bygningsstatiske Meddelelser, 49(1978), 1, (in Danish with English summary).

2) *Idem:* "A lifetime analysis of cracked linear viscoelastic materials – with special reference to wood", Conference of Int. Union of Forestry Research Organizations (IUFRO), Borås, Sweden 1982. Proc., Chalmers Univ. Technology, Dept. Struct. Eng., Gothenburg, Sweden 1982, 152 - 178.

3) *Idem:* "On the Mechanical Behavior of Materials with Interacting Cracks", Conference of Int. Union of Forestry Research Organizations (IUFRO), Borås, Sweden 1982. Proc., Chalmers Univ. Technology, Dept. Struct. Eng., Gothenburg, Sweden 1982, 141 - 149.

4) Madsen, Borg: 'Structural Behavior of Timber', Timber Engineering Ltd., North Vancouver, B.C., 1992. Appendix by Nielsen, L. Fuglsang: 'The theory of wood as a cracked viscoelastic material'

5) Nielsen, L. Fuglsang: "Lifetime, Residual Strength, and Quality of Wood - and other viscoelastic materials". Holz als Roh- u. Werkstoff, 49(1991), 451-455.

6) *Idem:* 'On the influence of crack closure on strength estimates of wood', Holz als Roh- und Werkstoff, 62(2004), 81-87.

7) Madsen, B.: "Duration of Load Tests for Dry Lumber in Bending". Forest Products Journ., 23(1973): 21-28.

8) Wood, L.W.: "Relation of strength of wood to duration of stress". U.S. Forest Products Laboratory, Madison, Wisc., USA, Report No. R-1916(1951).

9) Madsen, B. and Johns, K. 1982: "Duration of Load Effects in Lumber, Part II: Experimental Data". Can. J. Civ. Eng., 9: 515-525. Johns, K. and Madsen, B.: "...Part I: A Fracture Mechanics Approach". Can. J. Civ. Eng., 9: 502-514. Madsen, B. and Johns, K.: ".... Part III: Code Considerations". Can. J. Civ. Eng., 9, 526-536.

10) Spencer, R.: "Rate of loading effect in bending for Douglas-fir lumber", First Int. Conf. on Wood Fracture, Banff, Alberta, Canada, 1978. Proc. Western Forest Products Lab., Univ. Brit. Columbia, Vancouver, Canada, 1979, pp. 259 - 279.

11) Foshi, R.O. and Barrett, J.D.: "Duration of load effects in Western Hemlock Lumber". Journ. Struct. Div., ASCE, 108(1982): 1494-1510.

12) *Idem:* "Duration of load effects, 2x8 spruce lumber in bending: Data analysis and modeling", Western Forintek, Canada, Spec. Publ., SP-47(1986).

13) Krebs, H.J.: "The influence of moisture content on the long-term mechanical properties of wood" (in Danish). Thesis, Build. Mat. Lab., Tech. Univ. Denmark, 1984.

14) Gerhards, C.C.: "Effect of grade on load duration of Douglas-fir lumber in bending", Wood and Fiber Sci. 20(1988): 146-161.

15) Hoffmeyer, P.: "Failure of wood as influenced by moisture and duration of load". Thesis, Dept. Environmental Science and Forestry, State Univ. of New York, Syracuse, N.Y., 1990.

16) Nielsen, L. Fuglsang: ‘On the influence of moisture and load variations on the mechanical behavior of wood’, working paper for the COST-E24 meeting in Ljubljana October 7-8, 2004.

17) *Idem:* "Fatigue-styrken af beton og lignende viskoelastiske systemer med ældning" (fatigue strength of concrete and similar viscoelastic systems with aging). Tech. report 196 (1989). Lab. for Bygningsmaterialer, Danmarks tekniske Højskole.

18) Sørensen, N. Bo.: "Udmattelse af højstyrkebeton" (fatigue strength of high strength concrete), PhD.-thesis, Instituttet for Bygningsteknik, Ålborg Universitetscentrum, Ålborg 1993.

19) Nielsen, L. Fuglsang: "Power Law Creep as Related to Relaxation, Elasticity, Damping, Rheological Spectra and Creep Recovery - With Special Reference to Wood", Conference of Int. Union of Forestry Research Organizations (IUFRO), Xalapa, Mexico, December 1984. Proc., Build. Mat. Lab., Techn. Univ. Denmark 1985, 181 - 204.

20) *Idem:* “Modified Dugdale cracks and fictitious cracks - subjected to sub-critical loading”, Bygningsstatiske Meddelelser, 70(1999), 85-106.

21) *Idem:* "Lifetime of wood as related to strength distribution", in "Reliability-Based Design on Engineered Wood Structures", (ed. J. Bodig), Kluwer Academic Publishers, Dordrecht, The Netherlands, 1992.

22) Spencer, R.A. and Madsen, B.: "Duration of load tests for shear strength". Conference of Int. Union of Forestry Research Organizations (IUFRO), Xalapa, Mexico, December 1984. Proc., Build. Mat. Lab., Techn. Univ. Denmark 1985, 225 - 243.

23) Nielsen, L. Fuglsang: ’Træs udmattelse – Prediktion af træbjælkers levetid og reststyrke” (Fatigue of Wood - Prediction of lifetime and residual strength for wood beams), Tech. report, LBM 294/1993, 65 pp, Dept. of Civil Eng., Technical University of Denmark. Final report on fatigue loading of Glue-lam in research program, ‘Advanced Materials based on Straw and Wood’, sponsored by the Ministry of Trade and Industry, Denmark.

24) Dugdale, D.S.: "Yielding of steel sheets containing slits". J. Mech. and Phys. of Solids, 8(1960), 100 - 104. thesis